# 3.7 The Distributive Property

Any successful sports team must practise. Often a practice begins with a warm-up. Suppose the coach tells her players to run the perimeter of the field. How can you use algebra to write an expression for the total distance? Is there more than one way to do this?

James claims, "I can calculate the perimeter of the field by using the formula $P = 2(l + w)$."

Sylvie replies, "That's not right, James. The correct formula is $P = 2l + 2w$."

Who is correct, James or Sylvie? Can they both be right?

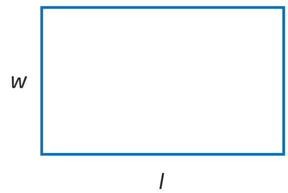
## Investigate

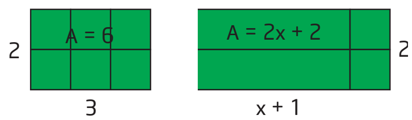
**Tools**
- algebra tiles

### How can you simplify algebraic expressions involving multiplication?

The area of a rectangle can be expressed as the product of its width and length:

$A = w \times l$

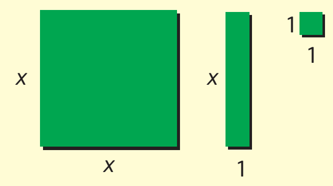


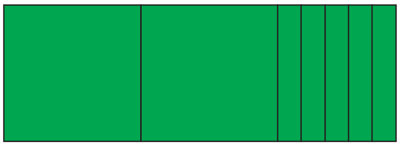
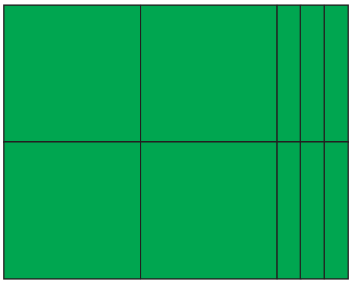

$A = 2 \times 3$
$= 6$

$A = 2(x + 1)$
$= 2x + 2$

The dimensions of algebra tiles:



**1.** Copy and complete the table. Use algebra tiles to help.

| Rectangle | Width | Length | Area | Equation $w \times l = A$ |
|---|---|---|---|---|
|  | $2$ | $x + 1$ | $2x + 2$ | $2 \times (x + 1) = 2x + 2$ |
|  | $x$ | | $x^2 + 2x$ | |
|  | | | | |
|  | $2x$ | $2x + 3$ | | |
| Create your own | | | | |

**2.** Look at the last column. Describe how you can start with the expression on the left side of the equation and get the expression on the right side.

**3.** Use the technique from step 2 to multiply the following.

**a)** $4(x + 3)$

**b)** $x(2x + 7)$

**c)** $3x(x + 2)$

**4.** **Reflect** Create an example and use it to explain how you can multiply a monomial by a polynomial.

Suppose that you have three dimes and three nickels.

The total amount can be expressed different ways:

$$\begin{aligned}&3(10) + 3(5)\\ =\ &30 + 15\\ =\ &45\end{aligned}$$

$$\begin{aligned}&3(10 + 5)\\ =\ &3(15)\\ =\ &45\end{aligned}$$

Whether you use 3(10) + 3(5) or 3(10 + 5), the result is 45¢. What does this tell you about these two numeric expressions?

Some algebraic expressions cannot be simplified using the second method. Reflect on the situations when you cannot simplify an expression. Suppose you add two nickels to a pile of nickels. The value of the nickels can be modelled by $5(x + 2)$, where $x$ is the number of nickels in the original pile.

$5(x + 2)$

You cannot add $x + 2$ because they are not like terms. However, you can multiply the 5 by both terms in the binomial:

$$\begin{aligned}&5(x + 2)\\ =\ &5(x) + 5(2)\\ =\ &5x + 10\end{aligned}$$

This is called the **distributive property**.

**distributive property**

- $a(x + y) = ax + ay$

## Example 1 Apply the Distributive Property

Expand.

a) $3(x + 2)$

b) $-5(4m - 3)$

c) $(2y + 5)(-4)$

d) $2(5a^2 - 7a + 2)$

**Literacy Connections**

When you apply the distributive property, you are **expanding** an expression.

### Solution

a)
$$\begin{aligned}&3(x + 2)\\ =\ &3(x) + 3(2)\\ =\ &3x + 6\end{aligned}$$

To apply the distributive property, I need to multiply both terms in the brackets by 3.

**b) Method 1: Show the Distributive Step**

$$
\begin{aligned}
&-5(4m - 3)\\
&= -5(4m + (-3))\\
&= (-5)(4m) + (-5)(-3)\\
&= -20m + 15
\end{aligned}
$$

When I distribute a negative term, I must follow the integer rules for multiplication.

**Remember to keep the negative sign when you are distributing.**

**Method 2: Apply the Distributive Step Mentally**

$$
\begin{aligned}
&-5(4m - 3)\\
&= -20m + 15
\end{aligned}
$$

$(-5) \times 4m = -20m$
$(-5) \times (-3) = 15$

**c)** The monomial is usually written before the polynomial, but sometimes it appears after. Apply the distributive property in the same way.

$$
\begin{aligned}
&(2y + 5)(-4)\\
&= 2y(-4) + 5(-4)\\
&= -8y + (-20)\\
&= -8y - 20
\end{aligned}
$$

**d)** You can also apply the distributive property to trinomials.

$$
\begin{aligned}
&2(5a^2 - 7a + 2)\\
&= 10a^2 - 14a + 4
\end{aligned}
$$

The distributive property can be applied to polynomials with any number of terms.

## Example 2 Distribute Variables

Simplify using the distributive property.

**a)** $x(x - 3)$ **b)** $p(p^2 - 2p + 1)$

### Solution

**a)**

$$
\begin{aligned}
&x(x - 3)\\
&= x(x) + x(-3)\\
&= x^2 - 3x
\end{aligned}
$$

To multiply variables, I need to use the exponent laws. The product rule is $x^a \times x^b = x^{a+b}$.

**b)**

$$
\begin{aligned}
&p(p^2 - 2p + 1)\\
&= p^3 - 2p^2 + p
\end{aligned}
$$

I need to use the product rule.
$p \times p^2 = p^{(1+2)}$
$= p^3$

The distributive property allows you to simplify complicated expressions.

## Example 3 Expand and Simplify Expressions

Expand and simplify.

**a)** $-4(x + 3) + 2(2x - 1)$

**b)** $3m(m - 5) - (2m^2 - m)$

**c)** $\frac{1}{2}(2w - 6) - \frac{2}{3}(9w - 6)$

### Solution

**a)**
$$\begin{aligned} &-4(x + 3) + 2(2x - 1) \\ &= -4(x) - 4(3) + 2(2x) + 2(-1) \\ &= -4x - 12 + 4x - 2 \\ &= -4x + 4x - 12 - 2 \\ &= -14 \end{aligned}$$

Apply the distributive property first to remove brackets. Then, collect like terms.

**b) Method 1: Add the Opposite Polynomial**

$$\begin{aligned} &3m(m - 5) - (2m^2 - m) \\ &= 3m(m - 5) + (-2m^2 + m) \\ &= 3m^2 - 15m - 2m^2 + m \\ &= 3m^2 - 2m^2 - 15m + m \\ &= m^2 - 14m \end{aligned}$$

$-2m^2 + m$ is the opposite of $2m^2 - m$.

**Method 2: Distribute −1**

$$\begin{aligned} &3m(m - 5) - (2m^2 - m) \\ &= 3m(m - 5) + (-1)(2m^2 - m) \\ &= 3m^2 - 15m - 2m^2 + m \\ &= m^2 - 14m \end{aligned}$$

Multiplying a polynomial by −1 produces the opposite polynomial: $(-1)(2m^2 - m) = -2m^2 + m$

**c)**
$$\begin{aligned} &\frac{1}{2}(2w - 6) - \frac{2}{3}(9w - 6) \\ &= \frac{1}{\cancel{2}_1}(\cancel{2}^{1}w) + \frac{1}{\cancel{2}_1}(\cancel{-6}^{-3}) - \frac{2}{\cancel{3}_1}(\cancel{9}^{3}w) - \frac{2}{\cancel{3}_1}(\cancel{-6}^{-2}) \\ &= 1(1w) + 1(-3) - 2(3w) - 2(-2) \\ &= w - 3 - 6w + 4 \\ &= -5w + 1 \end{aligned}$$

## Example 4 Nested Brackets

Expand and simplify.

$3[2 + 5(2k - 1)]$

### Solution

When simplifying expressions involving nested brackets, begin with the inner brackets and work your way outward.

$$
\begin{aligned}
&3[2 + 5(2k - 1)] \\
&= 3(2 + 10k - 5) \\
&= 3(10k - 3) \\
&= 30k - 9
\end{aligned}
$$

Simplify the inner brackets before distributing the 3.

## Key Concepts

- The distributive property allows you to expand algebraic expressions:
  $a(x + y) = ax + ay$
  - When distributing, multiply the monomial by each term in the polynomial.
  - Multiply numerical coefficients.
  - Apply exponent laws to variables.

### Literacy Connections

When you *distribute* a handout in class, you give one copy to each student. Similarly, when you use the distributive property, the term in front of the brackets is multiplied by, or given to, each term inside the brackets..

## Communicate Your Understanding

**C1** Consider the expression $3(x + 5)$. Why do you need to use the distributive property? Why can you not just add the terms in the brackets first?

**C2** Explain the distributive property. Create an example to support your explanation.

**C3** Dmitri simplified the following expression, as shown:

$$
\begin{aligned}
&3x(x + 2) \\
&= 3x + 2x \\
&= 5x
\end{aligned}
$$

**a)** Explain the errors in Dmitri's solution and how to correct them.

**b)** How can you prove that Dmitri's answer is not correct?

**C4** Explain why the following simplification is incorrect.

$$
\begin{aligned}
&(x^2 - 3x + 1)(-5) \\
&= x^2 - 3x - 4
\end{aligned}
$$

**C5** Describe the steps you must follow to simplify the expression $3[x + 2(x - 1)]$.

## Practise

*For help with questions 1 to 3, see Example 1.*

**1.** Which expression shows $-3(x + 5)$ expanded?

**A** $-3x + 15$ **B** $-3x + 5$

**C** $-3x - 8$ **D** $-3x - 15$

**2.** Use algebra tiles, virtual algebra tiles, or a diagram to model and simplify each expression.

**a)** $2(x + 3)$ **b)** $x(x + 4)$

**c)** $3(x + 1) + 2(x + 5)$

**3.** Expand, using the distributive property.

**a)** $4(x + 2)$ **b)** $5(k - 3)$

**c)** $-2(y + 1)$ **d)** $-8(2 - d)$

**e)** $5(2t - 3)$ **f)** $-(4y - 5)$

*For questions 4 and 5, see Example 2.*

**4.** Expand.

**a)** $y(y - 4)$ **b)** $r(r + 5)$

**c)** $x(2x - 5)$ **d)** $q(-4q + 8)$

**e)** $z(-3z + 2)$ **f)** $m(-m - 5)$

**5.** Expand.

**a)** $2b(3b - 5)$ **b)** $3v(8v + 7)$

**c)** $-4w(3w - 1)$ **d)** $-6m(-m - 5)$

**e)** $2q(-4q + 3)$ **f)** $-3d(-d + 2)$

**6.** Expand using the distributive property.

**a)** $(n - 5) \times 4$ **b)** $(2p + 4) \times 9$

**c)** $(7m + 6) \times (-4)$ **d)** $(7 + c)(3c)$

**e)** $(3 - 6w)(-2)$ **f)** $(4k + 7)(-3k)$

**7.** Expand.

**a)** $2(a^2 + 5a + 3)$ **b)** $-3(2n^2 - 8n + 5)$

**c)** $4k(k^2 + k - 3)$ **d)** $-5h(3h^2 - 7h - 2)$

**e)** $(x^2 - 5x + 2)(-3)$ **f)** $(2y^2 + 3y - 1)(4y)$

*For help with question 8, see Example 3.*

**8.** Expand and simplify.

**a)** $3(x + 2) + 4(x - 5)$ **b)** $-4(y + 1) + 2(2y - 3)$

**c)** $2(u + v) - 3(u - v)$ **d)** $4(w - 2) - 2(2w + 7)$

**e)** $-3(a + b) - 2(a - b)$ **f)** $2(p - q) + 2(-p + q)$

*For help with question 9, see Example 4.*

**9.** Expand and simplify.

**a)** $3[x + 2(x - 4)]$ **b)** $5[2(y - 1) - 3]$

**c)** $3[2k - (2 + k)]$ **d)** $4[-3(r - 5) + 2r]$

**e)** $2[-h - 2(h - 1)]$ **f)** $-3[2(p + 2) - 3p]$

## Connect and Apply

**10.** A computer repair technician charges \$50 per visit plus \$30/h for house calls.

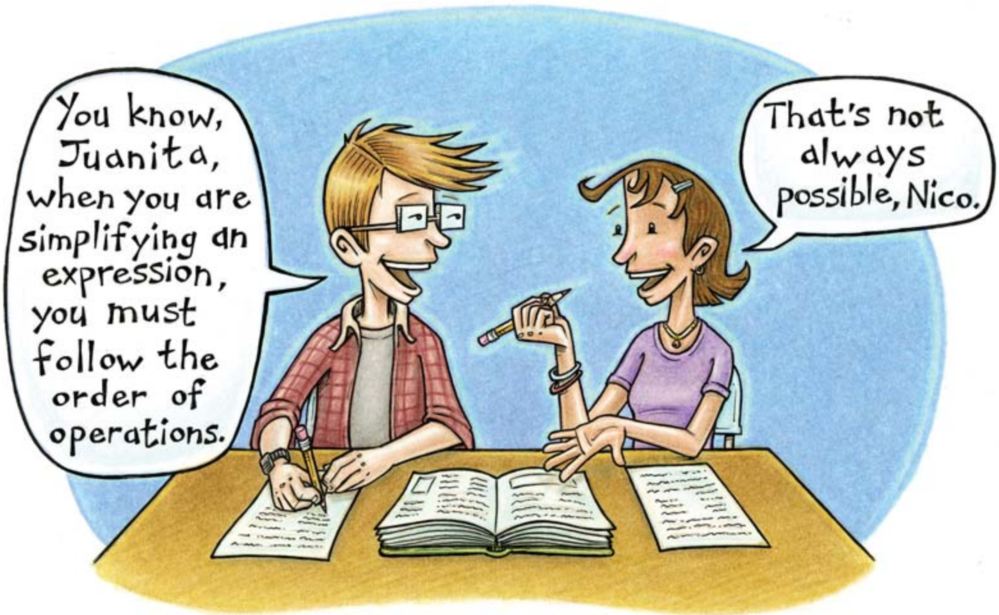
**a)** Write an algebraic expression that describes the service charge for one household visit.

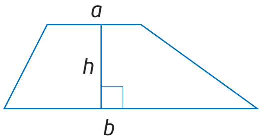
**b)** Use your expression to find the total service charge for a 2.5-h repair job.

**c)** Suppose all charges are doubled for evenings, weekends, and holidays. Write a simplified expression for these service charges.

**d)** Use your simplified expression from part c) to calculate the cost for a 2.5-h repair job on a holiday. Does this answer make sense? Explain.

**11.** Who is right? Use examples to explain your answer.



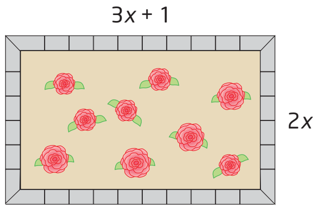



12. The formula for the area of a trapezoid is

$$A = \frac{1}{2}(a + b)h$$

Apply the distributive property twice to write this formula in another way.

13. A garden has dimensions as shown.



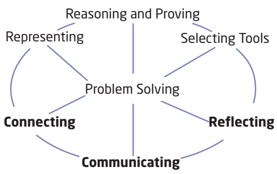

a) Find a simplified expression for the perimeter.

b) Find a simplified expression for the area.

c) Repeat parts a) and b) if both the length and width are tripled.

d) Has this tripled the perimeter? Justify your answer.

e) Has this tripled the area? Justify your answer.




14. Is the distributive property true for numerical expressions (expressions without variables)? Make a prediction. Then, create some numerical expressions and explore this. Describe any conclusions that you reach.

15. Expand and simplify.

a) $2(x - 3) + 3(x + 5)$

b) $3(k - 4) - 2(k + 1)$

c) $0.2p(p - 5) + 0.4p(3p - 2)$

d) $-4h(h + 2) + h(2h - 3)$

e) $5j(j - 3) - (2j^2 - 3j)$

f) $-0.7w(2w - 3) - 0.6w(w + 3)$

g) $3(y - 2) - 2(4 - 2y) + (6 - 7y)$

h) $4k(k - 3) - 2(k^2 - 3k + 4) - (k^2 - 5)$

16. Expand and simplify.

a) $\frac{1}{3}(3a + 2) + \frac{1}{4}(4a - 2)$

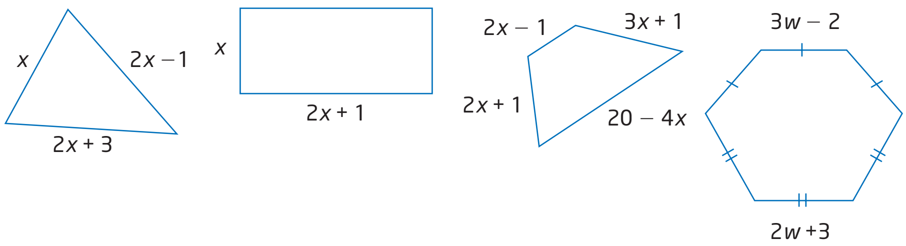
b) $\frac{1}{2}(x - 2y) + \frac{1}{3}(3y - 2x)$

c) $\frac{2}{3}(3m - 2) - \frac{3}{4}(8m - 2)$

d) $-\frac{1}{4}(4u - 3v) - \frac{3}{5}(6u - 10v)$

## Achievement Check

**17. a)** Find a simplified expression for the perimeter of each figure. Use algebra tiles if you wish.




**b)** A rectangle has length $2x - 1$ and width $8 - 2x$. What is unusual about the perimeter?

**c)** For what value of $x$ is the rectangle in part b) also a square?

## Extend

**18.** Expand and simplify.

**a)** $2m[m - 3(m + 2)]$

**b)** $-3p[2p(p + 4) + p^2]$

**c)** $2[3 + 2(x - 6)] + 3[-2(x - 5) + 8]$

**d)** $-3[2 - (y - 5)] - 4[3(y + 1) - (6 - y)(-2)]$

**19.** How do you distribute a binomial? Consider the following product of two binomials:

$(x + 3)(2x - 2)$

Distribute the $(x + 3)$ to each term in the second binomial to form an expression consisting of two products:

$(x + 3)(2x) + (x + 3)(-2)$

**a)** Simplify this expression. Check to see that your result appears to be equivalent to the original expression, by substituting values into both expressions.

**b)** Make up your own question similar to this and simplify it.

**c)** Can you see an easier way to expand the product of two binomials? Find and explain an easier method.

**Making Connections**

You will multiply binomials in grade 10 and beyond as a method of simplifying polynomial expressions.

**20.** Expand and simplify. Check to see if your result is correct. If you need a hint, look at the method from question 19.

$(k + 1)(k^2 - 2k - 1)$

**21. Math Contest** If $\left(\frac{1}{2}\right)^x = 0.015\ 625$, determine the value of $x$.

# Computer Algebra Systems

A computer algebra system (CAS) is a type of calculator or computer software that can perform algebraic functions. They are sometimes called symbolic manipulators. You can investigate many of the skills that you have learned in this chapter using CAS systems.

**Tools**

- TI-89 calculators

**Technology Tip**

Before you start doing calculations, clear the calculator's memory.

Press 2nd F1 for [F6] to display the **Clean Up** menu.

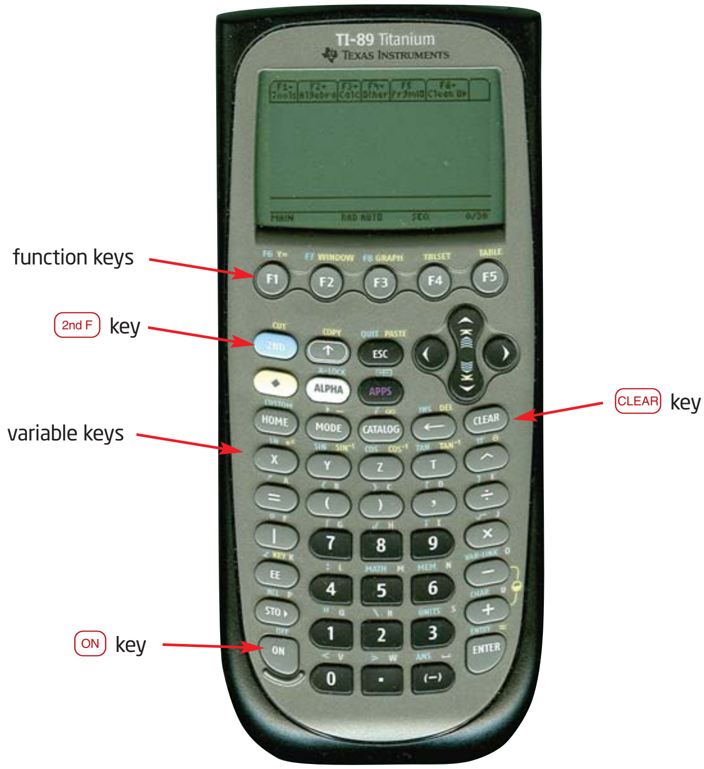
Select **2:NewProb**. Press ENTER.

**Technology Tip**

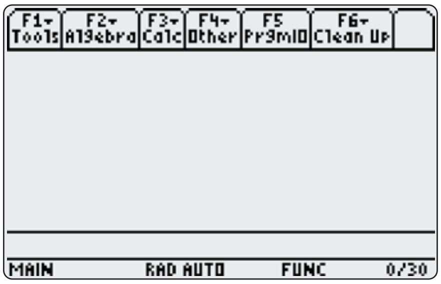
Note that you use a different button for leading negative signs (-), as in the −5 in question 2 part c), than the subtraction button −.

## Investigate A

### How can you learn about a CAS?



1. Press the ON key to turn the calculator on. If you do not see the Home screen, as shown, press the HOME key.




2. Perform some basic calculations to get familiar with the keyboard and display. Follow the keystrokes shown.

   **a)** $3(6 - 4)^5$

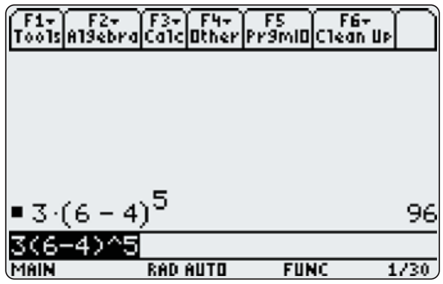
   3 × ( 6 − 4 ) ^ 5 ENTER

   **b)** $\pi\,(2.5)^2$

   2nd ^ × 2.5 ^ 2 ENTER

   **c)** $-5 - \sqrt{9}$

   (-) 5 − 2nd × 9 ) ENTER




3. Explore the Function keys at the top of the keypad.

   **a)** Press F1 to see the TI-89 **Tools** menu. Identify at least three commands that you recognize from working on a computer. Use the up and down cursor keys to see more commands.

   **b)** Press F2 to see the algebra functions that the TI-89 can perform. Identify two commands that you recognize from this chapter and two that you do not recognize.

   **c)** You can also use the left and right cursor keys to see the different function menus. Scroll over to see the menus of **F3** and **F4**. You will learn about these functions in future mathematics courses.

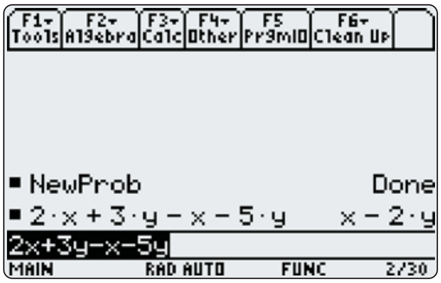
   **d)** Clean up the home screen:
   - Press 2nd [F6] to display the **Clean Up** menu.
   - Select **2:NewProb**.
   - Press ENTER.

# Use Technology

## Investigate B

### How can you use a CAS to simplify and expand expressions?

**1.** Collect like terms for the expression $2x + 3y - x - 5y$.
- Type the expression $2x + 3y - x - 5y$.
- Press ENTER.




**2.** Use the CAS to collect like terms for each expression.

**a)** $3y - 2x + 5x - 3y$

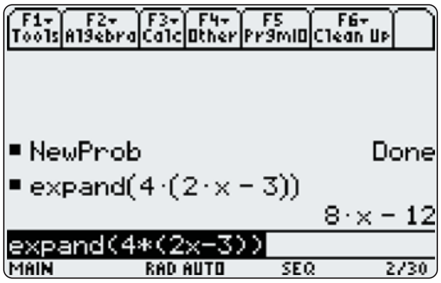
**b)** $x^2 + 3x - 2 - 6x + 3x^2$

**c)** Create your own expression.

**Technology Tip**

Note the nested brackets required in this command line. If you get an error message, check that you have the same number of open and close brackets, and that they are placed correctly.

**3.** Expand $4(2x - 3)$ using the distributive property.
- Press F2.
- Select **3:expand(**.
- Type the expression.
- Press ).
- Press ENTER.




**4.** Use a CAS to simplify using the distributive property.

**a)** $-2z(3z - 5)$

**b)** $3(4x - 7y) - 2(3x - 2y)$

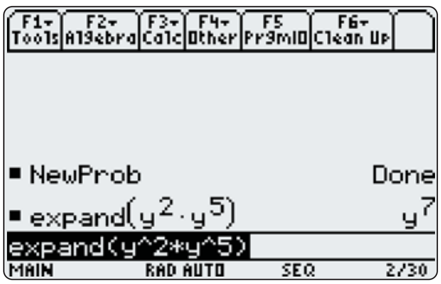
**c)** Create your own expression.

## Investigate C

### How can you verify the exponent laws?

**1. a)** Use a CAS to illustrate the product rule of exponents by entering the expression $y^2 \times y^5$.

**b)** Create two of your own examples to illustrate the product rule using a CAS.




**2.** You can also evaluate expressions involving powers, such as $y^2 \times y^3$ when $y = 3$.

Press Y ^ 2 × Y ^ 3 ÷ Y = 3.

Use this feature to evaluate

**a)** $y^3 \times y^4$ when $y = 4$ **b)** $\frac{z^8}{z^5}$ when $z = -5$

**c)** $\frac{x^4 \times x^7}{x^5}$ when $x = \frac{2}{3}$

**3.** Design your own method for verifying either the quotient rule or the power of a power rule for exponents. Describe your steps, keystrokes, and screen outputs.

## Investigate D

### How can you explore an unknown function?

Many of the functions of the CAS will be unknown to you at this level. Several of them you will learn about in future mathematics courses, but in this section you will have the opportunity to experiment and learn a little about some of these "mystery" functions.

**1. a)** Clear the home screen. Press 2nd F1 for [F6] to display the **Clean Up** menu. Select **2:NewProb** and press ENTER.

**b)** Use the **Expand** command to simplify the expression $3(x + 4)$.

**c)** Factor the result:
- Press F2.
- Select **2:Factor(**.
- Type $3x + 12$.
- Press ).
- Press ENTER.

What does the **Factor** function do?

**d)** Write down the result. What do you notice?

**2.** Repeat step 1 for each expression.
**a)** $y(2y - 3)$ **b)** $5z(2z - 1)$

**3.** Explain what the **Factor** command seems to do. How does it seem to be related to the **Expand** command?

**4. a)** Repeat step 1 for the expression $2(2x + 6)$.

**b)** Is the factored result what you expected? Explain.

**c)** Create a few examples of your own. Describe what you discover.

**5.** Explore some of the unknown features of the TI-89. Try to figure out what one of them does. Write a brief report of your findings.

**Making Connections**

You will learn about factoring in grade 10.